Solving Linear Equations in One Variable

Worked Examples

Solve the equation $30x - 1 = 22x + 5$

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Our goal is to isolate the *x* on one side. We'll do that by adding (or subtracting) quantities from both sides and multiplying (or dividing) both sides by nonzero numbers, simplifying after each step, to get a string of equivalent equations. When we've finished, we hope the last one will be *x* = some number. You can choose to do these in any order that pleases you – as long as you always keep the equation in balance.

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$$
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30x - 1 &= 22x + 5 \\
-22x &\quad -22x \\
8x - 1 &= 5
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Now I want to get all the numbers on the other side. First, I will add 1 to both sides:

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+1 &\quad +1 \\
8x &= 6
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Finally, I want just *x* alone, not 8*x*. So I'll divide both sides by 8:

$$
\begin{aligned}
8x &= 6 \\
\div 8 &\quad \div 8 \\
x &= \frac{6}{8} = \frac{3}{4}
\end{aligned}
$$

Solve the equation $30x - 1 = 22x + 5$

Recapping all the steps in one string of equations:

$$30x - 1 = 22x + 5$$
$$8x - 1 = 5$$
$$8x = 6$$
$$x = \frac{3}{4}$$

Because we ended up with the form x = number, we know our equation has exactly one solution, and we can read the solution directly. The set of all numbers that make the original equation true, the solution set, is {3/4}.

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It's a good idea to check by substituting our solution back into the original equation.

$$30\left(\frac{3}{4}\right) - 1 \overset{?}{=} 22\left(\frac{3}{4}\right) + 5$$

Simplifying the left side: $30\left(\frac{3}{4}\right) - 1 = \frac{45}{2} - 1 = \frac{43}{2}.$

Simplifying the right side: $22\left(\frac{3}{4}\right) + 5 = \frac{33}{2} + 5 = \frac{43}{2}.$

Since both sides are the same, 3/4 is a solution to the equation.

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Add 6*x* to each side: $70 = 12$

This equation is never true.

That means that our original equation has no solutions.

The solution set is { }.

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2c = 20
c = 10.

If you make 10 calls, your bill will be $35.

(Check: $10 + $2 for each of the 10 calls does make $35, yes.)

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So you should make no more than 10 calls this month.

(ps – your bill will still be more than $35, because we forgot about the taxes.)

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Translate into an equation. Let d be the number of Democrats in the House of Representatives. Then d is 77 more than the rest, so the rest is d – 77. Together, the Democrats and all the others, make up the 435 seats, so

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The Democrats held 256 seats in the House of Representatives.

(Check: If the Democrats had 256 seats, then the other 179 seats were something else. And, yes, 256 is 77 more than 179.)