# Solving linear equations in 1 variable

## Why do we need to be able to do this?

One of the reasons we study algebra is to systematically answer questions like "What should I charge for the coffee in order to make the largest profit?" or "What angle of launch elevation will send the missile the furthest?" or "Which cell-phone plan is least expensive for me?" or "Should my factory make more skateboards or snowboards?" Questions like these can often be answered by finding solutions to equations. The simplest and most basic equations to solve are linear equations. Luckily, linear equations have many applications.

## What should you be able to do?

**Recognize a linear equation in 1 variable.** It's an equation (so it has an equals sign) It contains only one variable (for example, $x$), and that variable is only to the first power – no squares, no square roots, $x$ is not in the denominator, and so on. A linear equation in one variable can always be rewritten as $ax + b = c$, where $a$, $b$, and $c$ are real numbers. (And $a$ is not zero.)

**Know when to solve an equation.** Solving an equation means finding all the solutions to the equation. A solution to an equation is a number or set of numbers that makes the equation true. The solution set is the set of all solutions to the equation; the list of all the solutions written inside set brackets.

In a math class, there are two kinds of problems where you need to solve equations. Some are meant for practicing algebraic techniques, and the instructions will say "Solve for $x$" or "Find all solutions to ..." Others are applied problems, and the story will ask you to find a particular solution.

**Solve a linear equation in 1 variable.** An equation is a balance – any time we do something to one side of the equation, we must do the same thing to the other side so that we remain in balance. There are only a small handful of things we can do to an equation that will give an equivalent equation (one that has exactly the same solutions).

We can add (or subtract) the same quantity to both sides of the equation. This can be a number, or it can be an expression involving variables – as long as it's the same on both sides. Adding the same quantity on both sides of the equals sign will give an equivalent equation (that means the same numbers will make it true).

We can multiply or divide both sides of the equation by a nonzero number, a constant. (Remember that dividing is just multiplying by the reciprocal.) Be careful about multiplying by a variable – if it should turn out to be zero, you would not end up with an equivalent equation.

Using these rules, we try to rewrite our linear equation (that is, find an equivalent equation) in a form that looks like $x$ = number.

There are three things that can happen as we try this.

- We find an equivalent equation that looks like $x$ = number. At that point, we can read the solution directly; the number is the only solution. Depending on the problem, we may have to write the solution in a complete English sentence, or as a solution set.
- We find that our equation is equivalent to some equation that is always true, such as $4 = 4$, or $x = x$. In this case, we know that our equation is always true, that every real number is a solution, and that the solution set is the set of all real numbers. This solution set can be written as $\mathbb{R}$ or $(-\infty, \infty)$.
- We find that our equation is equivalent to some equation that is never true, such as $4 = 1$. In this case, we know that our equation is never true, that no real number will be a solution, and that the solution set is the empty set. This solution set can be written as $\{\ \}$ or $\varnothing$.