Proportional Reasoning

Why do we need to be able to do this?

Proportional reasoning compares ratios to answer questions.

We can use proportional reasoning to solve some questions directly, such as which size of laundry detergent is the cheapest per load, or what the dimensions of the model car should be.

We can also use proportional reasoning to estimate answers and check answers to problems involving more complicated algebra techniques.

In geometry, the concept of **similar figures** comes up frequently. Two figures are similar if they have exactly the same shape, but not necessarily exactly the same size. You may hear similar figures described as "proportional." That's because corresponding lengths of similar figures have a constant ratio.

What should you be able to do?

Translate back and forth between ratios and fractions. Proportional reasoning relies on ratios. A key idea is that every ratio can be written as a fraction, and every fraction can be thought of as a ratio.

Example: I make just 2/3 as much as my husband – this is thinking about it as a fraction. Thinking about it as a ratio, I might say – I make $2 for every $3 he makes.

Use proportional reasoning to answer questions. Proportional reasoning involves comparing ratios.

Example: Your laundry detergent comes in three sizes – a little 50-ounce bottle on sale for $7.99, a medium 100-ounce bottle for $13.99, and a big 150-ounce bottle on sale for $17.99. Which is the best deal?

Proportional reasoning to the rescue here – we can compare the price per ounce and see which is least. The phrase "price per ounce" tells us exactly which ratios we should compare – price divided by ounces.

Little bottle: $7.99/50 ounces is about $0.16 per ounce.
Medium bottle: $13.99/100 ounces is about $0.14 per ounce.
Big bottle: $17.99/150 ounces is about $0.12 per ounce.

The big bottle is the best deal.

By the way, you could just as easily answer this question using ounces per dollar – the bottle with the most ounces for each dollar would be the best deal. Would you get the same answer?

Use proportional reasoning with estimation. Proportional reasoning is a great tool to quickly estimate an answer. You can also use proportional reasoning with estimation to check the answers to algebra problems.

Example: I know I can drive about 300 miles on a full tank of gas. I plan to visit my grandmother, a round trip of about 200 miles. How much of a tank will I probably use? 200 is 2/3 of my full range, so I expect to use about 2/3 of a tank.

Example: The formula for converting Fahrenheit temperatures into Celsius temperatures is $C = \frac{5}{9}(F - 32)$. I converted 70ºF to 7ºC. Did I do that right?

5/9 is about ½, and 32 is about 30, so my answer should be about ½ (70 – 30) = ½ ·40 = 20 degrees. I must have made a mistake. (The correct answer is about 21º.)

Worked Examples:

Example: I know I can drive about 300 miles on a full tank of gas. My gas tank looks like it is about 1/4 full. How many more miles can I drive?

If I have a quarter tank, I can drive about a quarter of my full range, so about 75 miles.

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By the way, you could just as easily answer this question using ounces per dollar – the bottle with the most ounces for each dollar would be the best deal. Would you get the same answer?

Bonus question. Suppose you have a coupon for $4 off any size bottle of your detergent. If you use your coupon, which bottle will have the smallest price per ounce?

Little bottle: $3.99/50 ounces is about $0.08 per ounce.
Medium bottle: $9.99/100 ounces is about $0.10 per ounce
Big bottle: $13.99/150 ounces is about $0.09 per ounce.

The coupon makes the little bottle the best deal per ounce on this trip.

Example: My iTunes library has about 9000 songs, for a total of 32.42 GB. Unfortunately, my iPod has a usable capacity of only 27.8 GB. About how many of my songs can I fit on my iPod?

The iPod can hold about $\frac{27.8}{32.42} \approx 0.8575$ of my library. Of my 9000 songs, then, it can hold about $0.8575 \cdot 9000 = 7700$ songs. Check: The iPod said it would hold about 7500 songs, so that's right. I need a bigger iPod.

Example: Certain model cars have a 1/43 ratio, which means that each inch of length on the model corresponds to 43 inches of length on the real car. I have a doll who measures 1.5 inches tall. Will she look the right size next to one of these model cars?

If each inch of the doll's height corresponds to 43 inches on a real person, she will represent a person who is 1.5 times 43 inches, or 64.5 inches tall. She'd be about 5 foot 5 inches, which seems reasonable for a real person.