PROBLEMS USING COMBINATIONS AND PROBABILITY

License Plate Problems

Each of the following problems assumes that a state's license plates consist of a certain number of letters followed by a certain number of numbers.

1) How many different plates can be made with one letter followed by one number?

There are 26 choices of letters. Each of these 26 letters can be followed by one of 10 different digits -- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Therefore there are 26 x 10 = 260 different plates which can be made.

2) How many different plates can be made with two letters followed by three numbers?

There are 26 choices for the first letter. For each of these letters, there are 26 choices for the second letter. There are therefore 26X26 = 676 possible pairs of letters. (Note that a repeat letter, such as DD, is allowed and so we do not use ${}_{26}P_2$.)

You must now consider the three numbers. There are 10 possibilities for the first digit, 10 possibilities for the second, and 10 possibilities for the third. This means that there are 10 x 10 x10 = 1000 different numbers. (Note that this is simply saying that there are 1000 numbers between and including 000 and 999.)

Combining these results, it follows that there are 676 x 1000 = 676,000 different license plates possible.

3) What is the probability that I will have a license plate with my initials, MR, followed by a number which ends in 4?

The probability of any event occurring is (the number of ways the event can occur) divided by (the number of possible outcomes). Since MR can occur in only one way and there are 676 possible pairs of letters, the probability of getting MR is $\frac{1}{676}$. There are ten different choices for the last digit, but only one of these choices is a 4. Therefore the probability of getting a number which ends in 4 is $\frac{1}{10}$.

Selecting the letters and the numbers are independent events, and so the probability of receiving a license plate which has the letters MR followed by a number which ends in 4 is $\frac{1}{676} \times \frac{1}{10} = \frac{1}{6760}$.

Card Selection Problems

1) What is the probability of drawing a heart from an ordinary deck of cards?

There are 13 ways to select a heart and a total of 52 possible selections. The probability of selecting a heart is therefore $\frac{13}{52}=\frac{1}{4}$.

2) What is the probability of drawing two hearts from a deck of cards?

As shown above, the probability that the first card will be a heart is $\frac{1}{4}$. There are now only 12 hearts left out of 51 cards. Therefore, the probability that the second card will be a heart is $\frac{12}{51}$. The probability that both events will occur is $\frac{1}{4}\times\frac{12}{51}=\frac{1}{17}$.

Washington State Gambling Problems

1) State Lottery

You select 6 numbers out of 49. The State then selects 6 numbers. To win, all six of your numbers must be the same as the 6 the State has selected. What is the probability of winning?

There is only one way of winning. The number of possible outcomes is ${}_{49}C_6 = 13983816$.
Therefore, the probability of winning is $\frac{1}{13983816} \approx .0000000715112$.

2) Pick Six -- big winner

This is a form of Keno. There are 80 possible numbers. You select 6 numbers. The State then selects 20 numbers. To win, your 6 numbers must be included in the 20 numbers selected by the State. What is the probability of winning?

First you must compute the number of ways of winning. Your 6 numbers must be included in the 20 winning numbers. This means that there are ${}_{20}C_6 = 38760$ ways to win. Next compute the number of possible selections. Since you select 6 from 80 numbers, there are ${}_{80}C_6 = 300{,}500{,}200$ possible selections.

The probability of being a big winner is $\frac{38{,}760}{300{,}500{,}200} \approx .000129$.

3) Pick Six -- lesser winner

What is the probability of having exactly 5 of your selected numbers be drawn by the State?

The chances of having 5 of your numbers be on the winning list is ${}_{20}C_5$ and the probability of having the remaining number not be selected is ${}_{60}C_1$. Therefore, the number of ways you can be a lesser winner is ${}_{20}C_5 \times {}_{60}C_1 = 930240$.

The probability of being a lesser winner is $\frac{930{,}240}{300{,}500{,}200} \approx .003096$.