Working with Functions

Why do we need to be able to do this?

The notion of a function is one of the most powerful in mathematics. It's a surprisingly simple idea.

One of the first things you think about mathematics is "there's only one answer." That's not always true. But it's the basic idea behind functions – a function is a reliable process. If you give a function the same information, it will always give you the same answer.

In higher mathematics (and the sciences that depend on them), this reliable nature of functions is what makes them so important. And if we can think of our situation as a function, many powerful tools become available to us.

What should you be able to do?

Recognize a function, no matter what the context. A *function* is a correspondence between two sets that assigns to each element of the first set **exactly one** element of the second set. The first set, the set of inputs, is called the *domain*. The second set, the set of outputs, is called the *range*.

Functions do not have to have anything to do with numbers. The key point is those words "**exactly one**." That makes them predictable, and that's the reason they're so important.

Functions can come to you in many different forms. (Have we seen this before?) A function can be expressed as a list of ordered pairs or as a table (numerically). A function can be expressed as a formula or algebraic rule that tells what to do to the input to produce its output (algebraically). A function can be expressed as a graph (graphically) – to find the output (y) for a particular input (x), you read the graph. A function can be expressed in English as well.

No matter what form a relationship appears, it isn't a function unless it has that "exactly one" feature. If you have a table of numbers, you need to make sure that no input has more than exactly one output. If you have a graph, you need to be sure that no x-value has more than one y-value associated to it (the vertical line test). If the correspondence is described in English, you need to be sure that no input has more than one answer associated to it. If you see functional notation, you usually get the "exactly one" feature for free (see below).

Understand and be able to use functional notation. Functional notation is a way to write a function with an algebraic rule. This is the way most students think about functions (which may be why so many people become confused about functions). For example, the function that associates to each number its square could be written:

$$f(x) = x^2$$

This is read "*f* of *x* equals *x* squared."

The *f* here is the name of the function. You'll often see *f* used for function, because *f* is the first letter in the word "function." But any letter or combination of letters would be fine. In fact, it's a good idea to pick a letter that will remind you of what you're doing.

The parentheses here do not denote multiplication. They're read aloud as "of." The fact that they're right next to the name of the function tells you that this is a function, and you should look inside them to see what the variable will be.

The *x* here is the variable name. Again, *x* is very commonly used, but there's nothing magic about it. You could use any letter or symbol that you like. The point is to look within the parentheses to see what letter is there, because that's what will stand for the input in the rule.

The algebraic stuff on the right hand side of the equals sign is the rule. This is the part that tells you what to do with your input. Your input goes exactly in place of the variable (which you identified right above). This rule says "take the input and square it."

As long as you don't have any choices in your algebraic rule, like a ± sign, or more than one possible rule, a correspondence written this way will always be a function. Algebra has that property built in – you always get the same answer when you plug in the same input.

Understand what $y = f(x)$ means. This familiar expression is actually just the connection between the algebraic idea and graphical idea of a function. You can think of "$y = f(x)$" as instructions for graphing a function from a formula – for every *x*, you get the *y*-coordinate of the point on the graph by evaluating the formula for *x*.

Find domains (and sometimes ranges) of functions. The domain is the set of all allowable inputs for a function. If the function is given to you as a table or graph, you can read those inputs off. If the function is given to you as a graph, the domain is the portion of the *x*-axis that is in the "shadow" of the graph. If the function is described in English, so there is a real setting, the setting may further restrict the domain (for example, lengths must be positive).

If the function is given as an algebraic rule, you can find the domain by throwing out any inputs that don't make sense, that are undefined. For example, you can't divide by zero or find a real square root of a negative number. Once you've thrown out any problem inputs, the domain is all the other real numbers.

Sometimes you may be able to find the range of a function. The easiest way to do this is to graph the function and look for the "shadow" along the *y*-axis. Unfortunately, for many functions you need to have had calculus before you can be sure what the range is.