Completing the Square

Worked Examples

Complete the square: $x^2 + 9x + 5$

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The leading coefficient is already 1. Taking half of 9 gives us 9/2, so the square we want is $\left(x + \frac{9}{2}\right)^2 = x^2 + 9x + \frac{81}{4}$.

Now we add and subtract 81/4, and regroup:

$$x^2 + 9x + 5 = x^2 + 9x + \left(\frac{81}{4} - \frac{81}{4}\right) + 5 = \left(x^2 + 9x + \frac{81}{4}\right) - \frac{81}{4} + 5$$

$$= \left(x + \frac{9}{2}\right)^2 - \frac{61}{4}$$

Complete the square: $4x^2 - 16x + 14$

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First, let's factor out the leading coefficient – we'll include the 4 at the end.

$$4x^2 - 16x + 14 = 4\left(x^2 - 4x + \frac{7}{2} \right)$$

Now we complete the square as before with $x^2 - 4x + \frac{7}{2}$.

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Now we complete the square as before with $x^2 - 4x + \frac{7}{2}$.

One half of –4 is –2, so the square we want is $(x-2)^2 = x^2 - 4x + 4$.

Add and subtract the 4 and regroup:

$$x^2 - 4x + \frac{7}{2} = x^2 - 4x + (4-4) + \frac{7}{2} = (x^2 - 4x + 4) - 4 + \frac{7}{2}$$

$$= (x-2)^2 - \frac{1}{2}.$$

Finally, we bring back the 4 we factored out at the beginning:

$$4x^2 - 16x + 14 = 4\left((x-2)^2 - \frac{1}{2}\right) = 4(x-2)^2 - 2.$$