COMBINATIONS

A combination is one of a collection of objects where order is not considered. For example, if you are dealt a hand of five cards, it makes no difference which order you received the cards.

Introductory problem

Suppose we have five people on a committee and we wish to select two people to serve as co-chairs. How many different pairs of people can be selected?

If we represent the five committee members by the letters A, B, C, D, and E, we can list all possible choices.

AB
AC
AD
AE
BC
BD
BE
CD
CE
DE

By counting the number of pairs, we see that there is a total of 10 different combinations.

Notation

Several notations are used to express the idea of n objects taken r at a time.

All of the following mean the same thing. $\;{}_nC_r \qquad C(n,r) \qquad C^n_r \qquad \binom{n}{r}$

Computation

You can either use a calculator or the following formula: $\;{}_nC_r = \dfrac{n!}{r!(n-r)!}$.

We can use this formula on the "Introductory Problem" as follows:

$${}_5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3 \times 2}{2(3 \times 2)} = 10$$

Harder Problem

Given a deck of 52 cards, how many 5-card hands can be dealt?

$${}_{52}C_5 = \frac{52!}{5!(52-5)!} = \frac{52!}{5!\,47!} = \frac{52 \times 51 \times 50 \times 49 \times 48 \times 47!}{5 \times 4 \times 3 \times 2 \times 47!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2} = 2,598,960$$

This means that there are 2,598,960 different 5-card hands.